Use sigma notation to write the series $\frac{65}{3} - \frac{62}{6} + \frac{59}{12} - \cdots + \frac{35}{3072}$. ← ARITHMETIC $d=-3$ ← GEOMETRIC $r=-2$ **SCORE: _____ / 20 PTS**

Show clearly the algebra used to find the upper limit of summation (ie. not just by counting).

$$\sum_{n=1}^{11} \frac{65-3(n-1)}{3\cdot(-2)^{n-1}} = \sum_{n=1}^{11} \frac{68-3n}{3\cdot(-2)^{n-1}}$$

$68-3n = 35$

IF $n=11$

Find parametric equations for the ellipse with $(-1,-5)$ and $(-1,11)$ as foci, and $(1, 3)$ as one endpoint of the minor axis.

SCORE: _____ / 20 PTS

F $\cdot (-1,11)$

C $\cdot (-1,3)$ $\cdot (1,3)$

F $\cdot (-1,-5)$

$a^2 = 2^2 + 8^2$

$a^2 = 68$

$a = 2\sqrt{17}$

$x = -1 + 2\cos\theta$

$y = 3 + 2\sqrt{17}\sin\theta$

Find the coefficient of $x^{12}y^{46}$ in the expansion of $(7x^3-5y^2)^{27}$. **SCORE: _____ / 20 PTS**

Your final answer may use $+, -, \times$ and positive powers. It may **NOT** use $\cdots$, !, $\div$, negative exponents **NOR** fractions.

It does **NOT** need to be simplified into a single number. **Show all work as demonstrated in lecture.**

$$\sum_{i=0}^{27}\binom{27}{i}(7x^3)^{27-i}(-5y^2)^i$$

$$=\sum_{i=0}^{27}\binom{27}{i}7^{27-i}\,x^{3(27-i)}(-5)^i y^{2i}$$

$3(27-i)=12$ AND $2i=46$

IF $i=23$

$$\binom{27}{23}7^4(-5)^{23}$$

$$=\frac{27\cdot 26\cdot 25\cdot \cancel{24}}{\cancel{4\cdot 3\cdot 2\cdot 1}}\cdot 7^4(-5)^{23}$$

$$=27\cdot 26\cdot 25\cdot 7^4(-5)^{23}$$

Consider the sequence defined recursively by $a_n = n^2 - 2a_{n-1}, \quad a_1 = 2$. SCORE: _____ / 10 PTS

Find the first 4 terms of the sequence, and **write them as a list**.

$a_2 = 2^2 - 2a_1 = 4 - 4 = 0$

$a_3 = 3^2 - 2a_2 = 9 - 0 = 9$

$a_4 = 4^2 - 2a_3 = 16 - 18 = -2$

$2, 0, 9, -2$

Last year was a good year for the Lee's family business. During January 2012, revenues were $19,173 and operating costs were $5,418.

SCORE: _____ / 30 PTS

<u>NOTE: You must show the use of the relevant sequence and/or series formulae to earn full credit for the following problems.</u>

[a] If revenues each month were 1.5% higher than revenues the previous month, what was the total revenue from January 2012 to August 2012 ?

$$\frac{19173(1.015^8-1)}{1.015-1} = \$161,682.82$$

[b] If operating costs each month decreased by a fixed amount (in dollars) from the previous month, and operating costs in August 2012 were $4,144, what were the total operating costs from January 2012 to **<u>December</u>** 2012 ?

$$4144 = 5418 + 7d$$

$$d = -182$$

$$\frac{12}{2}(2(5418) + 11(-182)) = \$53,004$$

Find rectangular equations corresponding to the parametric equations $x = \dfrac{t}{2-t}$, $y = \dfrac{t+3}{t-3}$.

SCORE: _____ / 20 PTS

Write y as a function of x.

$$x(2-t) = t$$
$$2x - tx = t$$
$$2x = t + tx$$
$$2x = t(1+x)$$
$$t = \frac{2x}{1+x}$$

$$y = \frac{\frac{2x}{1+x} + 3}{\frac{2x}{1+x} - 3} \cdot \frac{1+x}{1+x}$$

$$y = \frac{2x + 3(1+x)}{2x - 3(1+x)}$$

$$y = \frac{5x+1}{-x-3} \text{ OR } -\frac{5x+1}{x+3}$$

Prove by mathematical induction: $\sum_{i=1}^{n}(2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ for all integers $n \geq 1$. **SCORE: _____ / 30 PTS**

HINT: The algebra in the proof is much easier if you use factoring.

$$\sum_{i=1}^{1}(2i-1)^2 = 1^2 = 1 = \frac{1(1)(3)}{3}$$

ASSUME $\sum_{i=1}^{k}(2i-1)^2 = \frac{k(2k-1)(2k+1)}{3}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$\sum_{i=1}^{k+1}(2i-1)^2 = \sum_{i=1}^{k}(2i-1)^2 + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= (2k+1)\left[\frac{k(2k-1)}{3} + 2k+1\right]$$

$$= (2k+1)\left[\frac{2k^2-k+6k+3}{3}\right]$$

$$= \frac{(2k+1)(2k^2+5k+3)}{3}$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

SO, BY MATHEMATICAL INDUCTION,

$$\sum_{i=1}^{n}(2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

FOR ALL INTEGERS $n \geq 1$